



Mark Scheme

Summer 2023

Pearson Edexcel GCE

Advanced Subsidiary Level

Further Mathematics (8FM0)

Paper 23 Further Statistics 1

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 40.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.

### 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.  
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.
7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Qu	Scheme		Marks	AO
1	$E(X) = 0 \times r + k + 2 \times \frac{k}{2} + 3 \times \frac{k}{3} + 4 \times \frac{k}{4} \quad \underline{\text{or}} \quad 4k$		B1	2.1
	$E(X^2) = k + 2^2 \times \frac{k}{2} + 3^2 \times \frac{k}{3} + 4^2 \times \frac{k}{4} \quad \underline{\text{or}} \quad k + 2k + 3k + 4k \quad \underline{\text{or}} \quad 10k$		M1	1.1b
	$\sqrt{\text{Var}(X)} = E(X) \Rightarrow E(X^2) = 2[E(X)]^2 \underline{\text{or}} \quad 10k - (4k)^2 = (4k)^2$		M1	1.1b
	$10k = 32k^2 \Rightarrow k = \frac{5}{16}$		A1	1.1b
	$\left[ \sum \text{probs} = 1 \Rightarrow \right] r + \frac{1}{12}(12k + 6k + 4k + 3k) = 1$ $\left[ \Rightarrow r + \frac{25}{12}k = 1 \right]$ $r = 1 - \frac{25}{12} \times \frac{5}{16} = \frac{67}{192}$		M1     A1	3.1a     1.1b
			(6)	
Notes				
	B1	for a correct expression for $E(X)$		
	1 <sup>st</sup> M1	for attempting $E(X^2)$ – at least 3 correct non-zero terms		
	2 <sup>nd</sup> M1	Use of $\text{Var}(X) = E(X^2) - [E(X)]^2$ to form an expression in $k$ with $\sqrt{\text{Var}(X)} = E(X)$ ft their $10k$ and their $4k$ Must use a consistent expression for $E(X)$ in both side of their equation		
	1 <sup>st</sup> A1	for a correct equation for $k$ (e.g. 2TQ) or $k = \frac{5}{16}$ or 0.3125. They may ignore or reject solution of $k = 0$ Allow for $r = 0.349$ or better		
	3 <sup>rd</sup> M1	for attempt at an equation in $r$ and $k$ using sum of probs. At least 4 terms correct in terms of $k$ or numerically using their value of $k$		
	2 <sup>nd</sup> A1	for $\frac{67}{192}$ or exact equivalents e.g. 0.3489583 Correct exact answer implies full marks		

Qu	Scheme		Marks	AO
2.(a)	$E(RR) = \frac{60 \times 60}{150}$ or $E(BB) = \frac{30 \times 27}{150}$ or $E(YY) = \frac{60 \times 63}{150}$		M1	1.1b
	$E(RR) = \underline{24}$ and $E(BB) = \underline{5.4}$ and $E(YY) = \underline{25.2}$		A1 (2)	1.1b
(b)	[E(BB) > 5 so no need for pooling] $v = (3 - 1)(3 - 1) = 4$		B1	3.1b
	Critical value: $\chi_4^2(5\%) = \underline{9.488}$		B1ft	1.1b
	(significant) evidence that the <u>colour of balls</u> is <b>not</b> independent )		B1ft (3)	2.2b
(c)	$O_i > E_i$ for pairings of the same colour <b>and</b> some children may be cheating when selecting the 2 <sup>nd</sup> ball so choice not independent (o.e.)		B1 (1)	3.5a
			(6)	
Notes				
(a)	M1	for at least one correct calculation seen		
	A1	for all 3 expected frequencies correct, must be exact.		
(b)	1 <sup>st</sup> B1	for 4 degrees of freedom		
	2 <sup>nd</sup> B1ft	for awrt 9.488 or ft their degrees of freedom for 5% cv		
	3 <sup>rd</sup> B1ft	for correct contextual conclusion mentioning “colour” and “independent” Must be consistent with their cv Allow associated instead of not independent		
(c)	B1	Stating or indicating that $O_i > E_i$ <b>and</b> a suitable reason for the lack of independence that would invalidate the modelling assumption. Such as: <ul style="list-style-type: none"><li>cheating/looking</li><li>The first ball being replaced by the child so more likely to be picked again</li></ul>		

Qu	Scheme	Marks	AO
3. (a)	[F = no of faults in A m <sup>2</sup> ] $F \sim \text{Po}(A \times 0.4)$	M1	3.3
	$[P(F = 0) \Rightarrow ] \ 0.0907 = e^{-0.4A}$	M1	3.4
	$A = \underline{6}$	A1	1.1b
		(3)	
(b)	[T = no. of tablecloths with no faults] $T \sim B(20, 0.0907)$	M1	3.3
	$P(T > 1) = 1 - P(T \leq 1)$	M1	1.1b
	$= 0.55276... = \text{awrt } \underline{0.553}$	A1	3.4
		(3)	
(c)(i)	[ $X \sim B(100, 0.0907)$ ]	M1	3.3
	$E(X) = 100 \times 0.0907 = \underline{9.07}$	A1	1.1b
	(ii) $\text{Var}(X) = 100 \times 0.0907 \times (1 - 0.0907) = 8.247351 \quad \text{awrt } \underline{8.25}$	A1	1.1b
		(3)	
(d)	$X \approx \sim \text{Po}(9.07) ;$	M1	3.4
	$P(X = 10) \approx 0.11947... \quad \underline{0.1195}$ or awrt <u>0.119</u>	A1	1.1b
		(2)	
(e)	$H_0: \lambda = 0.4 \quad (\text{or } \lambda = 12) \quad H_1: \lambda < 0.4 \quad (\text{or } \lambda < 12)$	B1	2.5
	[Y = no. of faults from new machine] $Y \sim \text{Po}(12)$	M1	1.1b/3.3
	$P(Y \leq 6) = 0.04582...$	A1	3.4
	[Significant] there is evidence to <u>support</u> the <u>claim</u>	A1	2.2b
		(4)	
(f)	$p - \text{value} = 0.04582.. \quad \text{awrt } \underline{0.0458}$	B1ft	1.2
		(1)	
		(16)	
Notes			
(a)	1 <sup>st</sup> M1	for selecting the correct model $\text{Po}(0.4A)$	
	2 <sup>nd</sup> M1	for a correct equation - may be implied by a correct answer with no incorrect working	
	A1	for $A = 6$ (or awrt 6.0)	
(b)	1 <sup>st</sup> M1	for selecting a correct model $B(20, 0.0907)$ used, or seen if only distribution in (b)	
	2 <sup>nd</sup> M1	for correctly interpreting “more than 1” to reach $1 - P(T \leq 1)$ . May be implied by A1	
	A1	for awrt 0.553	
(c)(i)	M1	for $X \sim B(100, 0.0907)$ used, or seen if only distribution in (c).	
	A1	May be implied by correct $E(X)$ or $\text{Var}(X)$	
		for 9.07	
(ii)	A1	for awrt 8.25 <b>SC - award M0A1A0 for:</b>	
		<ul style="list-style-type: none"><li>using <math>X \sim B(100, 0.4)</math> leading to <math>E(X) = 40</math>, <math>\text{Var}(X) = 24</math></li><li>using <math>X \sim B(100, p)</math>, <math>0 &lt; p &lt; 1</math> and <math>E(X) = 100p</math>, <math>\text{Var}(X) = 100p(1 - p)</math></li></ul>	
(d)	M1	for selecting the correct Poisson model – fit their answer to (c)(i)	
	A1	for 0.1195 or awrt 0.119	
(e)	B1	for both hypotheses correct in terms of $\lambda$ or $\mu$	
	M1	for selecting a suitable model. Sight or use of $\text{Po}(12)$ . May be implied by 1 <sup>st</sup> A1	
	1 <sup>st</sup> A1	for a correct probability must be 0.046 or better	
	2 <sup>nd</sup> A1	for a correct conclusion in context using “claim” or “rate of faults”	
(f)		B1ft for awrt 0.0458 o.e. e.g. 4.58% or fit their answer to 1 <sup>st</sup> A1 in (e)	



Qu	Scheme		Marks	AO
4. (a)	H <sub>0</sub> : No. of car <u>breakdowns</u> per month follows a <u>Poisson</u> distribution H <sub>1</sub> : No. of car <u>breakdowns</u> per month does not follow a <u>Poisson distribution</u>		B1 (1)	2.5
(b)	A <u>Poisson</u> distribution will assign some probability to (all) values greater than 5 (o.e.)		B1 (1)	2.4
(c)	Need $\lambda$ : $[\hat{\lambda} = ] \frac{0 \times 12 + 1 \times 11 + 2 \times 19 + 3 \times 14 + 4 \times 3 + 5 \times 1}{(12 + 11 + 19 + 14 + 3 + 1)}$ <u>or</u> 1.8  [Under H <sub>0</sub> $X \sim \text{Po}(1.8)$ $E_1 = 60 \times P(X=1) = 17.85(227\dots)$ $E_2 = 60 \times P(X=2) = 16.06(705\dots)$ $E_{\geq 5} = 60 - \sum_0^4 E_i = 2.18(43996\dots)$		M1  M1 A1 B1ft (4)	1.1b  3.4 1.1b 1.1b
(d)	$\frac{(11 - "17.85\dots")^2}{"17.85\dots"} = 2.6287\dots$ or $\frac{(14 - 9.64)^2}{9.64} = 1.97195\dots$  awrt 2.63 and awrt 1.97		M1  A1 (2)	1.1b 1.1b
(e)	Need to combine last two columns since $E_i$ are $< 5$ Degrees of freedom therefore $5 - 2$ since mean for Poisson estimated from $O_i$		B1 B1 (2)	1.1a 1.1a
(f)	$\chi^2_3(5\%) = 7.815$ (Not significant) insufficient evidence to <u>reject Anja's belief</u>		B1 B1 (2)	1.1b 2.2b
			(12)	
Notes				
(a)	B1	correct hypotheses mentioning “breakdowns” and “Poisson”. Po(1.8) o.e. is B0		
(b)	B1	for a suitable reason mentioning the Poisson distribution taking values beyond 5 e.g. sample space for Poisson being $[0, \infty)$		
(c)	1 <sup>st</sup> M1 2 <sup>nd</sup> M1 1 <sup>st</sup> A1 B1ft	for an expression for the mean with least 3 correct products and correct denominator M0 if they have found $\lambda$ by working backwards from $P(X=0) \Rightarrow 60e^{-\lambda} = 9.92$ for a correct method for finding $E_1$ or $E_2$ (implied by one correct value) for awrt 17.85 <u>and</u> awrt 16.07 for awrt 2.18, or using $60 - (\text{sum of their } E_i)$ or $60 \times (1 - P(X \leq 4))$		
(d)	M1 A1	for <u>either</u> correct expression (ft their $E_1$ ) for awrt 2.63 <u>and</u> awrt 1.97		
(e)	1 <sup>st</sup> B1 2 <sup>nd</sup> B1	for explaining need to pool columns since $E_i < 5$ allow candidates describing combining last three columns as long as they say $E_i < 5$ for mentioning mean/rate/parameter/ $\lambda$ estimated from $O_i$ and 2 constraints		
(f)	1 <sup>st</sup> B1 2 <sup>nd</sup> B1	for correct cv of 7.815 (or better) for correct conclusion in context mentioning “breakdowns” and “Poisson” Allow equivalent words for belief, such as “theory” etc. Must be consistent with their cv Allow Po(1.8) in conclusion instead of Poisson		

